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Statistical Characteristics of the Temporal Spectrum of Scattered Radiation in the Equatorial Ionosphere

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ABSTRACT

On the basis of the solution of the space-time characteristic system by the method of geometric optics using symbolic calculations, analytical and numerical simulation of the propagation of the ordinary and extraordinary radio waves in the conducting equatorial ionospheric plasma was made considering the anisotropy of plasma irregularities and non-stationary nature of propagation medium. Broadening of the spectrum and the displacement of its maximum contain velocity of a turbulent plasma flow and parameters characterizing anisotropic plasmonic structures. Statistical moments of both radio waves do not depend on the absorption sign and are valid for both active and absorptive random media. Temporal pulsations and conductivity of a turbulent ionospheric plasma have an influence on the evaluation of the spectrum-varying propagation distances travelling by these waves. The new double-humped effect in the temporal spectrum has been revealed for the ordinary wave varying anisotropy coefficient and dip angle of stretched plasmonic structures. From a theoretical point of view, the algorithms developed in this work allow effective modelling of the propagation of both radio signals in the equatorial conductive ionospheric plasma, considering the external magnetic field, inhomogeneities of electron density in-homogeneities, as well as non-stationary.

Keywords: Electromagnetic waves; Turbulence; Statistical characteristics; Waves propagation; Atmosphere; Ionosphere; Conductivity

1. Introduction

This paper reports a study of the features of radio wave propagation in the equatorial ionosphere of the Earth. The relevance of the work is determined by the wide use of decameter and decimeter radio

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waves to provide long-range radio communication, radio navigation, and radar, as well as to study the structure of the Earth's upper atmosphere. It should be noted that, despite numerous studies conducted by Ishimaru, Tatarskii, Gershman and Rytov [1-4], second-order statistical moments of scattered electromagnetic waves in the equatorial anisotropic ionosphere have not yet been studied enough and need both the development of new methods for modeling the propagation of electromagnetic radiation.

The ray theory is the main theoretical tool for describing the propagation of short radio waves in the Earth's equatorial ionosphere. It is based on the integration of stochastic differential equations, which describe the trajectories of rays in an in-homogeneous magnetized plasma. In addition, the calculation of the ray trajectories allows you to calculate the phase and take into account the absorption along the trajectories.

In the study by Jandieri et al. [5-10], peculiarities of the spatial power spectrum (SPS) of radio waves in the polar ionosphere were investigated by applying ray approximation. It has been established that randomly varying electron density and the ambient magnetic field and the anisotropy and dip angle of elongated inhomogeneities relative to the ambient magnetic field may increase the intensity of the frequency fluctuations of propagating radio waves in the terrestrial ionospheric plasma. Multiple scattered effects of waves are revealed more strongly at large scales with slowly varying time irregularities when secondary waves with close frequencies propagate in a narrow spatial angle near the direction of an initial wave.

The equatorial region of the terrestrial ionosphere is of great interest. Plasma bubbles, with a concentration greatly reduced relative to the background, are regularly observed in the equatorial region. These are relatively stable and rather large structures, the transverse size of which can reach hundreds of kilometers. When they rise, they stretch strongly along the magnetic force line, relying on their bases for the maximum of layer F2. The transverse dimension of the bubble measured along the parallel is about 450 km. The concentration inside the bud was reduced by about four times relative to the background. Statistical characteristics of propagating and scattered radio waves in the conductive equatorial ionosphere were not considered till now.

In reality, inhomogeneous media randomly vary both in space and in time. Harmonic waves scattered by the irregularities become nonharmonic. Spectral lines broaden. Generally, the propagation of a wave with a complex spectral structure continuously changes its spectrum. The features of the temporal spectrum in the polar ionosphere have been considered in the study by Jandieri et al. [11].

Important aspects of scattered ordinary (O-wave) and extraordinary (E-wave) radio waves propagating in the equatorial ionosphere, which vary slowly with position and time, are considered using the ray approximation. We will investigate the influence of the absorption on the second-order statistical moments of scattered radio waves in the collision-conductive ionospheric turbulent plasma using the stochastic transfer equation for the complex frequency based on the WKB (Wentzel-Kramers-Brillouin) method. Evaluation of the temporal (broadening and displacement of maximum) contains: the homogeneous external magnetic field, velocity of plasma flow; Hall, Pedersen, and longitudinal conductivities, anisotropy factor of electron density irregularities and the dip angle of prolate plasmonic structures with respect to the geomagnetic lines of forces.

Section 2 considers the ray approximation, spatial-temporal geometric optics (STGO), and ST rays in the equatorial ionospheric plasma. Expression of the complex refractive index of equatorial conducting magnetized plasma is obtained. A stochastic differential transfer equation for complex frequency fluctuation is given, describing the propagation of radio waves in plasma; statistical moments of the temporal spectrum describing scattering features of both ordinary and extraordinary radio waves are investigated. Numerical calculations are presented in Section 3 using the anisotropic spatial-temporal spectrum of electron density inhomogeneities and the velocity of turbulent plasma movement, applying the experimental data. Some conclusions and discussions are made in Section 4.

2. Materials and methods

Asymptotic and integrated methods are traditional instruments for studying radio waves' propagation in random media. Currently, the most developed and justified both theoretically and experimentally method of describing the mechanism of propagation of short (2-40 MHz) electromagnetic waves in the Earth's ionosphere is geometric optics—a method of approximate representation of wave fields in smoothly inhomogeneous media. The field of the monochromatic wave in a medium with a refractive index $N(\mathbf{r})$ in a scalar approximation is described by the Helmholtz equation which is given by Ishimaru, Tatarskii, Kravtsov et al. [1,2,12].

In the case of ionospheric propagation, the following inequality is usually satisfied: $l >> \lambda$ (λ is the wavelength of the wave). This inequality implies that only forward scattering is important in the random scattering process and the WKB solution is valid for the wave propagation. The phase satisfying the eikonal equation for each normal wave can be written as $c^2k^2 = \omega^2 N^2(\omega, \mathbf{k}, n)$, where $\mathbf{k}(\mathbf{r},t) = -\nabla\varphi$, $\omega(\mathbf{r},t) = \partial\varphi / \partial t$ are the local wave vector and the frequency, respectively, are spatial-temporal slowly-varying functions; $n(\mathbf{r},t)$ is a fluctuating component of the electron density of a turbulent plasma at the point $\mathbf{r}$; $N^2(\omega, \mathbf{k})$ is the complex refractive index, c is the speed of light.

Statistical analysis of a phase and its derivatives applying the WKB method is generally difficult, especially in a non-stationary medium when $\omega(\mathbf{r},t)$ is one of an unknown quantity. In quasi-monochromatic small-angle approximation, at smoothness electron density fluctuations $n(\mathbf{r},t) = n_0 + n_1(\mathbf{r},t)$, $|n_1| << n_0$, first term is a homogeneous component, the second one is spatial-temporal stochastic function describing electron density random variations. For complex refractive index $N(\mathbf{r},t) = N_0(\mathbf{r},t) - iN_1(\mathbf{r},t)$, we investigate statistical characteristics of the frequency fluctuations of electromagnetic waves propagating and scattering in the equatorial ionosphere. The imaginary component of the refraction index in inhomogeneous anisotropic conductive plasma within the framework of the ray approximation is connected with a complex wave vector. The real part of this vector is connected with both turbulent plasma inhomogeneity and refraction of waves, whereas the complex part of the inhomogeneous wave $(\mathrm{Im}\,\mathbf{k} \neq 0)$ describes the evolution of inhomogeneous waves including diffraction.

Let the ambient magnetic field is directed along the Y-axis. Components of the complex permittivity tensor in this case are: $\tilde{\varepsilon}_{xx} = \tilde{\varepsilon}_{zz} = \varepsilon_\perp - i(\tilde{\sigma}_\perp + sg)$, $\tilde{\varepsilon}_{xz} = -sæ\delta + \mathrm{i}(\tilde{\sigma}_H + æ)$, $\tilde{\varepsilon}_{yy} = (\varepsilon_\perp + p_0 u) - i(\tilde{\sigma}_\parallel + sv)$, $\tilde{\varepsilon}_{xx} = \tilde{\varepsilon}_{zz}$, $\tilde{\varepsilon}_{zx} = -\tilde{\varepsilon}_{xz}$, $\varepsilon_{xy} = \varepsilon_{yx} = \varepsilon_{yz} = \varepsilon_{zy} = 0$; here: $p_0 = v/(1-u)$, $g = p_0(1+u)/(1-u), \delta = 2/(1-u)$ $g_1 = (3-u)/(1-u)$, $æ = p_0\sqrt{u}$. Plasma parameters $v(\mathbf{r}) = \omega_p^2(\mathbf{r})/\omega^2$ and $u = (eH_0/m_e c\omega)^2$ contains the plasma frequency $\omega_p(\mathbf{r}) = \left[4\pi N_e(\mathbf{r})e^2/m_e\right]^{1/2}$ and the cyclotron frequency. Normalized conductivity tensor $\tilde{\sigma} = 4\pi\hat{\sigma}/k_0 c$ of ionospheric turbulent plasma for equatorial latitude was given in the study by Aydogdu et al. [13] contains the Hall's σ_H, Pedersen $\sigma_\perp$ and longitudinal $\sigma_\parallel$ conductivities:

$$\sigma_H = e^2 n_e \left(\frac{\omega_e}{m_e(v_e^2 + \omega_e^2)} - \frac{\omega_i}{m_i(v_i^2 + \omega_i^2)} \right),$$

$$\sigma_\perp = e^2 n_e \left(\frac{v_e}{m_e(v_e^2 + \omega_e^2)} + \frac{v_i}{m_i(v_i^2 + \omega_i^2)} \right),$$

$$\sigma_\parallel = e^2 n_e \left(\frac{1}{m_e v_e} + \frac{1}{m_m v_i} \right)$$

Here, $k_0 = \omega_0 / c$, e and m_e are the charge and mass of an electron, $v_{e,i}$ is the electron or ion collisional frequency with neutral molecules; ω_e and ω_i are the angular gyrofrequency of an electron and ion, respectively, m_i is the mass of ion.

Complex refractive index N of the conductive collision equatorial ionospheric plasma at $s \neq 0$, $\tilde{\sigma}_{ij} \neq 0$ and $s << \varepsilon_{ij}, \tilde{\sigma}_{ij}$ is as follows:

$$N^2(n,\omega) = \Gamma_0(n_0,\omega) + i\,\Gamma_1(n,\omega), \tag{1}$$

where: $\Gamma_0 = 1 - 2(T_1 T_0 - T_2\,\beta_2)/(T_1^2 + T_2^2)$,

$\Gamma_1 = 2(T_2 T_0 + T_1\,\beta_2)/(T_1^2 + T_2^2)$,

$T_1 = A \pm D_1$, $T_2 = R_1 \pm D_2$, $D_1 = \sqrt{(r_1 + B)/2}$, $r_1 = \sqrt{B^2 + C^2}$,

$D_2 = \sqrt{(r_1 - B)/2}$

$T_0 = p_0\, v\,(1-v) + \beta_1,\ \beta_1 = \Lambda_1 (\sin^2\theta - \varepsilon_{\parallel}) + \tilde{\sigma}_{\parallel}\tilde{\sigma}_{\perp}(1+\cos^2\theta - 2\varepsilon_{\perp}\varepsilon_{\parallel})$,

$C = \beta_6 - \beta_8,\ \beta_2 = \tilde{\sigma}_{\perp}\sin^2\theta + \tilde{\sigma}_{\parallel}\cos^2\theta - \Lambda_2 + 2\varepsilon_{\parallel}\varepsilon_{\perp}\tilde{\sigma}_{\perp} + \tilde{\sigma}_{\parallel}$
$(\varepsilon_{\perp}^2 - æ^2 - \Lambda_1)$,

$\Lambda_1 = \tilde{\sigma}_{\perp}^2 + \tilde{\sigma}_H^2 + 2\,æ\,\tilde{\sigma}_H,\ \ \beta_3 = \Lambda_1 \sin^2\theta + \tilde{\sigma}_{\parallel}\tilde{\sigma}_{\perp}(1+\cos^2\theta)$,

$a = 1 - p_0 (1 - u\cos^2\theta)\ \Lambda_2 = 2\varepsilon_{\perp}\tilde{\sigma}_{\perp}\sin^2\theta + (\varepsilon_{\parallel}\tilde{\sigma}_{\perp} + \varepsilon_{\perp}\tilde{\sigma}_{\parallel})$
$(1+\cos^2\theta)$,

$c = (1-v)\,[\,(1-v)^2 - u\,]/(1-u)$,

$b = \left[2(1-v)^2 - 2u + v\,u\,(1+\cos^2\theta)\right](1-u)^{-1},\ \ A = p_0\,[\,2(1-v)$
$-u\sin^2\theta\,]$,

$\beta_4 = \Lambda_2 - 2\,(\tilde{\sigma}_{\perp}\sin^2\theta + \tilde{\sigma}_{\parallel}\cos^2\theta)$,

$\beta_6 = 2\Lambda_2\left[\Lambda_1\sin^2\theta + \tilde{\sigma}_{\parallel}\tilde{\sigma}_{\perp}(1+\cos^2\theta) - b\right]$,

$B = (\beta_5 - \beta_7) + p_0^2\left[u^2\sin^4\theta + 4u\,(1-v)^2\cos^2\theta\right]$,

$\Lambda_3 = (\tilde{\sigma}_{\perp}\sin^2\theta + \tilde{\sigma}_{\parallel}\cos^2\theta)\left[2\varepsilon_{\parallel}\varepsilon_{\perp}\tilde{\sigma}_{\perp} + \tilde{\sigma}_{\parallel}(\varepsilon_{\perp}^2 - æ^2 - \Lambda_1)\right]$,

$\beta_5 = \Lambda_1^2\sin^4\theta + \tilde{\sigma}_{\parallel}^2\tilde{\sigma}_{\perp}^2(1+\cos^2\theta)^2 - \Lambda_2^2 - 2b\left[\Lambda_1\sin^2\theta + \tilde{\sigma}_{\parallel}\tilde{\sigma}_{\perp}\right.$
$\left.(1+\cos^2\theta)\right] + 2\Lambda_1\tilde{\sigma}_{\parallel}\tilde{\sigma}_{\perp}\sin^2\theta\cdot(1+\cos^2\theta)$,

$\beta_7 = 4\left[\Lambda_3 + a\,\varepsilon_{\parallel}\,(\Lambda_1 + 2\varepsilon_{\perp}\tilde{\sigma}_{\parallel}\tilde{\sigma}_{\perp})\right]$,

$\beta_8 = a\left[2\varepsilon_{\parallel}\varepsilon_{\perp}\tilde{\sigma}_{\perp} + \tilde{\sigma}_{\parallel}(\varepsilon_{\perp}^2 - æ^2 - \Lambda_1)\right] + (\tilde{\sigma}_{\perp}\sin^2\theta + \tilde{\sigma}_{\parallel}\cos^2\theta)$
$\left[c - \varepsilon_{\parallel}(\Lambda_1 + 2\varepsilon_{\perp}\tilde{\sigma}_{\parallel}\tilde{\sigma}_{\perp})\right]$

The upper sign is devoted to the O-wave, lower sign to the E-wave; θ is the angle between the wave vector $\mathbf{k}_0$ and the applied magnetic field $\mathbf{H}_0$. The refracting index of the equatorial ionosphere is complex. The absorption is always dissipative and represents a conversion of the wave energy into heat through the collision process. The presence of the geomagnetic field leads to birefringence and anisotropy. For collisionless and nonconductive turbulent plasma we obtain the well-known formula given in the study by Ginzburg et al. [14].

As is well known from the studies of Ishimaru, Tatarskii, Gershman, Rytov et al. [1-4], radio signals propagate in a non-stationary, turbulent plasma, the Doppler shift is small compared with the transmitter frequency, and the spectrum broadens. Quantitative estimation of the frequency fluctuations is important, as the broadening of a spectrum limits the resolution of a Doppler method studying the structure of the receiving signal. On the other hand, by measuring the width of the Doppler spectrum, it is possible to solve the reversal tasks by receiving information on statistical properties of turbulent plasma. The ratios connecting changes in frequency with the parameters of moving turbulent plasma irregularities make the application of statistical methods as a tool for solving direct and reverse problems of radio wave propagation in a non-stationary turbulent plasma necessary.

Neglecting polarization effects, in the geometrical optics approximation, the frequency of the radio wave satisfies the stochastic differential transport equation which is given in the study by Gavrilenko, Kravtsov et al. [15,16] for an arbitrary spatial-temporal dispersion:

$$\left(\frac{\partial}{\partial t} + \left(\mathbf{u}_{gr}\nabla\right)\right)\omega = -\frac{\omega u}{c}\sum\frac{\partial N}{\partial p_i}\frac{\partial p_i}{\partial t} \tag{2}$$

where: $N(\omega, p_i) = ck/\omega$ is the complex refractive index, $\mathbf{u} = (d\omega/d\mathbf{k})_{p_i}$ is the group velocity of the wave; p_i is an arbitrary parameter characterizing turbulent plasma. We assume that an incident plane wave propagates in the y direction. The eikonal equation describes virtual ray trajectories, which can be measured experimentally. Ray paths deviate toward an increase in the index of refraction. Radio waves propagating in the lower ionosphere, the ray path deviates towards the Earth; at the propagation of these waves through the upper ionosphere ray path deviates to the opposite direction.

Second-order statistical moment: the variance of the frequency fluctuations of radio signals is one of the important spectral characteristics that can be found by the measurement of random variations of a phase. It describes the broadening of the temporal spectrum in the ionospheric plasma. Applying equation (2) in a first approximation frequency fluctuation satisfies the stochastic transport differential equation:

$$\frac{\partial\omega_1}{\partial y} + \frac{q_0}{c}(1 + i\Lambda_0)\frac{\partial\omega_1}{\partial t} = -k_0(\Lambda_1 + i\ \Lambda_2)\frac{\partial n_1}{\partial t}, \tag{3}$$

where: $q_0 = N_0 + \omega_0\partial N_0/\partial\omega$, $\Lambda_0 = \dfrac{1}{q_0}\left(N_1 + \omega_0\dfrac{\partial N_1}{\partial\omega_0}\right)$,

$$\Lambda_1 = \frac{\partial N_0}{\partial n_0},\ \Lambda_2 = \frac{\partial N_1}{\partial n_0},$$

$$\Lambda_1 = \frac{1}{4N_0}\left[\left(\frac{\Gamma_0}{r_1} + 1\right)\frac{\partial\Gamma_0}{\partial n_0} + \frac{1}{r_1}\Gamma_1\frac{\partial\Gamma_1}{\partial n_0}\right],$$

$$\Lambda_2 = \frac{1}{4N_1}\left[\left(\frac{\Gamma_0}{r_1}-1\right)\frac{\partial \Gamma_0}{\partial n_0}+\frac{1}{r_1}\Gamma_1\frac{\partial \Gamma_1}{\partial n_0}\right]$$

is the group velocity of an unperturbed wave propagating along the y-axis in a conductive absorptive turbulent ionospheric magnetoplasma. In the anisotropic absorbing turbulent plasma, the direction of group speed and a wave vector cannot coincide. The energy flux of a wave propagating along the ray path at each point coincides with the group velocity v_{gr}. In the absence of the spatial dispersion, it also coincides with the direction of an average Poynting vector. However, the conductivity of turbulent plasma can lead to the opposite directions of Poynting's vector and a wave vector, and, hence, the group velocity will become negative.

For the solution of equation (3) we will apply the Fourier transform:

$$\omega_1(\mathbf{r},t)=\int_{-\infty}^{\infty} d\nu\,\Omega(\mathbf{r},\nu)\exp(i\nu t)\ .$$

Let a plasma slab having thickness L ($0<y<L$) contains electron density inhomogeneities with the linear scale l; a vacuum is at $y<0$ and the observation point is located out of a plasma slab $y>0$. We obtain:

$$\Omega(\tilde{\mathbf{n}}_\perp, y,\nu)=k_0\,\nu\,(\Lambda_2-i\,\Lambda_1)\exp\left[\frac{\nu}{c}\,q_0\,(\Lambda_0-i)\,y\right]\int_0^L d\zeta\ n_1(\tilde{\mathbf{n}}_\perp,\zeta,\nu)\exp\left[\frac{\nu}{c}\,q_0\,(-\Lambda_0+i)\,\zeta\right], \tag{4}$$

here $\tilde{\mathbf{n}}_\perp=\{x,z\}$. Spatial-temporal fluctuations of an instantaneous local frequency are caused by multiple scattering of radio waves propagating in a turbulent ionospheric plasma with a group velocity $V_{gr}=c\,(1-i\,\Lambda_0)/q_0$.

Non-stationarity is caused by moving turbulent plasma flow and spatial-temporal variations of electron density fluctuations. Monochromatic waves scattered on the electron density fluctuations become nonmonochromatic. Spectral lines broaden. Generally, waves with a complex spectral structure continuously change the spectrum. The spectrum of the receiving signal broadens and reaches its maximum during the Doppler shift due to irregularities and temporal fluctuations in electron density.

Statistical characteristics of the frequency fluctuation of radio waves propagating and scattering in a weakly absorptive $q_0<1$ equatorial ionosphere plasma at $L\gg 1$ can be written as:

$$\Delta_1=\pi\,(\Lambda_1^2+\Lambda_2^2)\,\frac{c\,k_0^2}{q_0\,\Lambda_0}\int_{-\infty}^{\infty} d\nu\,\nu\int_{-\infty}^{\infty} d\mathbf{k}_\perp\,W_n(\mathbf{k}_\perp, k_y,\nu)\left\{\exp\left(2\,\frac{q_0\,\Lambda_0}{c}\,y\,\nu\right)-\exp\left[2\,\frac{q_0\,\Lambda_0}{c}(y-L)\,\nu\right]\right\}\exp(i\,\mathbf{k}_\perp\,\boldsymbol{\rho}_\perp)\,, \tag{5}$$

$$\Delta_2=\pi\,k_0^2\,L\,(\Lambda_1^2-\Lambda_2^2)\int_{-\infty}^{\infty} d\nu\,\nu^2\int_{-\infty}^{\infty} d\mathbf{k}_\perp\,W_n(\mathbf{k}_\perp, k_y,\nu)\exp(i\,\mathbf{k}_\perp\,\boldsymbol{\rho}_\perp)\,, \tag{6}$$

where the asterisk means complex conjugation, $\mathbf{k}_\perp=(k_x,k_z)$.

$$W_n(\mathbf{k}_\perp,k_y,\nu)=\frac{1}{(2\pi)^4}\int_{-\infty}^{\infty} d\boldsymbol{\rho}_\perp\int_{-\infty}^{\infty} d\rho_y\int_{-\infty}^{\infty} d\tau\,V_n(\rho_\perp,\rho_y,\tau)\exp\left(i\,\mathbf{k}_\perp\,\boldsymbol{\rho}_\perp+i\,k_y\,\rho_y-i\,\nu\,\tau\right)$$

is an arbitrary spatial-temporal autocorrelation function of electron density fluctuations, $k_y=\nu q_0/c$. Temporal spectrum is calculated by the formula:

$$\Delta=\Delta_1+\Delta_2\,, \tag{7}$$

Broadening of the temporal power spectrum $\Delta\equiv<\omega_1^2>/\omega_0^2$ is an experimentally measurable parameter; $\Delta_1\equiv<\omega_1\omega_1^*>/2\omega_0^2$, $\Delta_2\equiv\mathrm{Re}<\omega_1\omega_1>/2\omega_0^2$; the pointed brackets indicate the ensemble average. Coherency impairment of the field of the receiving signal caused by phase fluctuations in the ionospheric plasma with large-scale inhomogeneities allows us to suggest that variance of the frequency fluctuations $<\omega_1^2>$ is also applicable at diffraction. At strong absorption the second term of equation (6) is much less than the first one.

Diffraction effects have a substantial influence on this second-order statistical moment only in Fraunhofer's zone $(y/k_0 l^2)\gg 1$.

This means that radio waves scattered under a big angle attenuate faster along the y axis.

3. Results

Experimental measurements of artificial large-scale electron density inhomogeneities generation in the ionospheric F_2 region by "SURA" heating facility by powerful radio waves were given in the study by Frolov et al. [17]. Transverse scale of these irregularities $l_\perp$ varies from a few meters up to tens of a hundred kilometers. Large-scale turbulent plasma irregularities along the sight beam on the satellite can have the sizes $l_\parallel = 30$ km, the drift velocity $30 \div 35$ m/s.

Observations of drift small-scale field-aligned inhomogeneities in the F ionosphere have shown in the study by Kvavadze et al. [18] that they have elongated shape, anisotropy factor varies from 1 to 3 moving with the drift velocity 40-100 meter/sec S-W direction. Small-scale irregularities with Gaussian spectrum are responsible for polarization fluctuations at the frequency band of 20-50 MHz.

Experimental measurements at Kingston, Jamaica show that field-aligned electron density inhomogeneities in the F-ionosphere are existing between altitudes 153 km and 617 km. Defiant scintillations of signals from the moving earth satellites were given in the study by Chen et al. [19]. Dip angle of these inhomogeneities relative to the ambient magnetic field was about 16°. Electron density irregularities are described by anisotropic Gaussian and power-law spectral functions. For F region large-scale sizes irregularities (~ 10 km) become unstable, and dissipate their energy by generating small-sized irregularities, as is the case in turbulence. In the equatorial region the large-scale irregularities are most likely produced by a convective electric field.

Analytical calculations are carried out using a spatial-temporal spectral function of electron density irregularities given in the study by Jandieri et al. [20]:

$$V_n(\mathbf{k},\nu) = \frac{\sigma_n^2}{16\pi^2} \frac{l_\parallel^3}{\chi^2 \left\{1 + l_\perp^2 \left[k_x^2 + (\nu q_0 / c)^2\right] + l_\parallel^2 k_z^2\right\}^2} \cdot \exp\left(-\frac{k_x^2 l_\perp^2}{4} - m_0^2 \frac{k_y^2 l_\parallel^2}{4} - m_1 \frac{k_z l_\parallel}{4} \nu T - m_2 \frac{\nu^2 T^2}{4}\right), \quad (8)$$

where: $m_0^2 = a_0 - 1/(p_2 \chi^2)$, $a_0 = (\Upsilon_1^2 / a_1) + p_2 (\Upsilon_2^2 / \Upsilon_0^2)$
$a_0 = (\Upsilon_1^2 / a_1) + p_2 (\Upsilon_2^2 / \Upsilon_0^2)$, $p_2 = (\sin^2\gamma_0 + \chi^2 \cos^2\gamma_0)/\chi^2$,
$a_1 = 1 + \zeta^2 (l_* / l_\perp)^2$, $m_2 = c_0 + p_2 q_0^2 \zeta_0^2 / \Upsilon_0^2$,
$l_* = l_\perp l_\parallel (l_\perp^2 \sin^2\gamma_0 + l_\parallel^2 \cos^2\gamma_0)^{-1/2}$, $\zeta = V_0 T / l_\parallel$,
$\Upsilon_0 = \left[1 - \zeta / (2 a_1)\right]^{1/2}$, $c_0 = (1/a_1) + p_2 \Upsilon_3^2 / \Upsilon_0^2$,
$\Upsilon_1 = (\chi^2 - 1)\zeta \sin\gamma_0 \cos\gamma_0 / \chi^2 p_2$, $\Upsilon_3 = \zeta / (a_1 p_2)$,
$m_1 = b_0 + p_2 \Upsilon_2 \zeta_0 q_0 / \Upsilon_0^2$, $b_0 = (\Upsilon_1 / a_1) - p_2 \Upsilon_2 \Upsilon_3 / \Upsilon_0^2$,
$\zeta_0 = (l_\parallel / cT)$, $\Upsilon_2 = (\chi^2 - 1)\sin\gamma_0 \cos\gamma_0 / (\sin^2\gamma_0 + \chi^2 \cos^2\gamma_0) - (\zeta \Upsilon_1 / a_1 p_2)$.

Diffusion processes in the ionosphere along and across directions relative to the external magnetic field lead to the anisotropy of elongated plasmonic structures. These inhomogeneities can be characterized by two anisotropy parameters: Anisotropy coefficient $\chi = l_\parallel / l_\perp$, which is the ratio of characteristic linear scales along and across the ambient magnetic field; and inclination angle γ_0 of electron density irregularities relative to the lines of forces of geomagnetic field; $T = l / V$ is the characteristic temporal scale of electron density fluctuations, $\nu_0 = 1/T$ is the frequency of the ionospheric irregularities. Drift velocity exceeds the velocity of the shape changes of the inhomogeneities themselves. If $V_0 = 0$ we obtain studies given by Jandieri et al. [8-10,20].

Substituting (8) into equations (5) and (6) we obtain the broadening of the temporal spectrum of scattered ordinary and extraordinary waves in the equatorial ionosphere:

$$\Delta_1 \approx \frac{\xi^2}{\chi} \frac{L}{l_\parallel} \left(\frac{\nu_0}{\omega_0}\right)^2 \frac{\sqrt{p_2}\,(\Lambda_1^2 + \Lambda_2^2)}{a_1 \Upsilon_0 t_0 \delta_1^3} \left\{ \frac{y}{L} - \left(\frac{y}{L} - 1\right) \exp\left[-4 \frac{\Lambda_0^2 q_0^2}{\delta_1^2} \zeta^2 \left(\frac{L}{l_\parallel}\right)^2 \left(2\frac{y}{L} - 1\right)\right]\right\} \cdot \left[1 + 4 m_2 \left(\frac{\delta_2}{\delta_1^2}\right)^2\right]^{-2} \exp\left[4 \frac{\Lambda_0^2 q_0^2}{\delta_1^2} \zeta^2 \left(\frac{y}{L}\right)^2 \left(\frac{L}{l_\parallel}\right)^2\right], \quad (9)$$

$$\Delta_2 \approx \frac{\xi^2 \delta_2^2}{\chi} \frac{L}{l_\parallel} \left(\frac{\nu_0}{\omega_0}\right)^2 \frac{\sqrt{p_2}\,(\Lambda_1^2 - \Lambda_2^2)}{a_1 \Upsilon_0 t_0 \delta_1^5} \left[1 + 4 m_2 \left(\frac{\delta_2}{\delta_1^2}\right)^2\right]^{-2} \exp\left(\frac{\delta_2^2}{\delta_1^2}\right), \quad (10)$$

where: $\delta_1 = \left[n_2 - (n_1^2 / n_0^2\right]^{1/2}$, $\delta_2 = 2\, q_0\, \zeta^2\, \Lambda_0\, (L / l_{\|})$ $\left[1-(y/L)\right]$, $m_2 = (n_1^2 / n_0^4) + \zeta^2 (q_0^2 / \chi^2)$, L is slab thickness.

Analyses show that frequency fluctuations grow faster than in nonabsorbing medium given in the studies by Gavrilenko, Kravtsov et al. [15,16] if distance travelling by wave in the turbulent plasma exceeds the distance of weak attenuation even at $y > L$.

It is essential to emphasize that second-order statistical moment (9) does not depend on a sign of the parameter ψ_0. So, the obtained result is valid for both absorbing, and active media. The reason for this effect is that, frequency perturbations in a medium with complex refractive index not only are transferred along Y axes with a group velocity given in the study by Kravtsov et al. [16], but also amplify. A similar effect can take place for an incident frequency-modulated wave in a homogeneous medium with a complex refractive index. The modulator in our case is a turbulent plasma layer.

Figures 1 and 2 illustrate ray trajectories of radio wave propagation in the turbulent anisotropic, inhomogeneous, non-stationary ionospheric plasma. Two groups of curves are highlighted. For higher frequencies propagating through the ionosphere, absorption is low. Lower frequencies are reflected from the layer. The characteristic maximum on curves is the reflection point. These rays spend a lot of time in the lower ionosphere and test active absorption. Absorption increases inversely in proportion to the frequency. Comparing the figures extraordinary wave is absorbed more than the ordinary one.

Figure 3 illustrate the behavior of the spectrum for different anisotropy factor. Analyses show that varying anisotropic parameters in the interval $3 \le \chi \le 18$ maximum, spectrum broadening increases and its maximum shifts to the left.

Curves describing the broadening of the temporal spectrum of O-wave scattered in the turbulent ionospheric plasma at an anisotropy factor $\chi = 3$ varying distance $0 \le (y/L) \le 10.5$ are plotted in **Figure 4**. Numerical calculations show that at $\gamma_0 = 17^0$ two humps arise; broadening of the first hump is very small. At $\gamma_0 = 19^0$ these humps have the same widths. Increasing the inclination angle up to $\gamma_0 = 21^0$ the second hump is disappeared. The same behavior of the curves is observed at $\chi = 6$, but the broadening of a second curve becomes substantially small at $\gamma_0 = 9$. Precursor in the temporal spectrum does not arise for E-wave.

Numerical calculations show that the broadening of the temporal spectrum of both O- and E-waves increases in proportion to the temporal parameter ν_0 / ω_0 and the distance propagating by radio waves in the equatorial ionosphere.

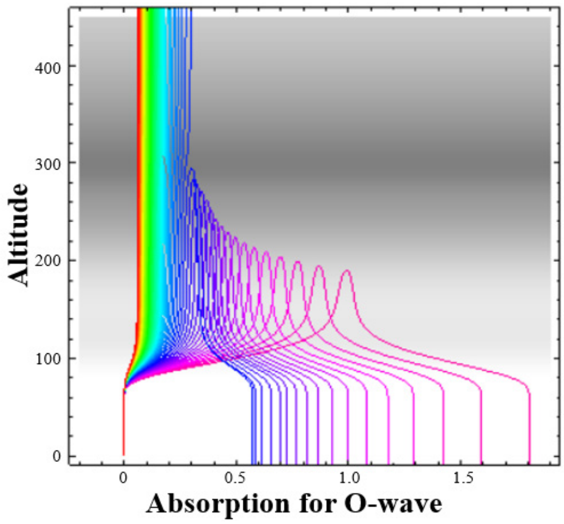

Figure 1. Altitude-absorption diagram for O-wave.

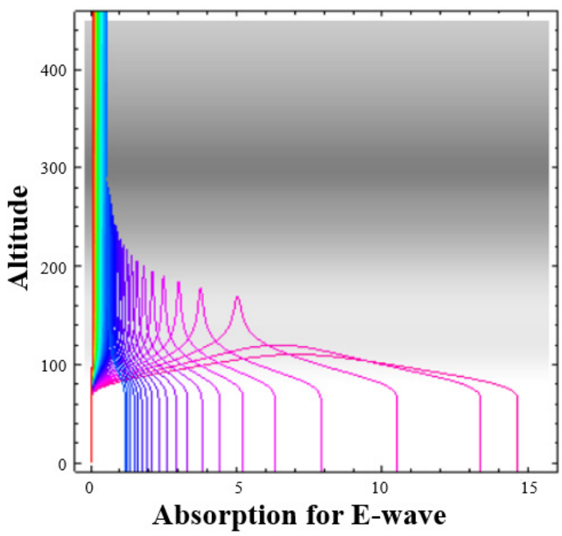

Figure 2. Altitude-absorption diagram for E-wave.

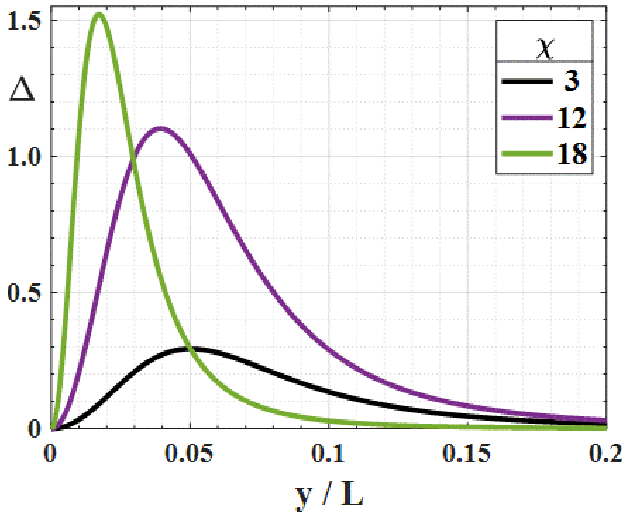

Figure 3. $\Delta - y/L$ diagram showing spectrum broadening for different anisotropy factor.

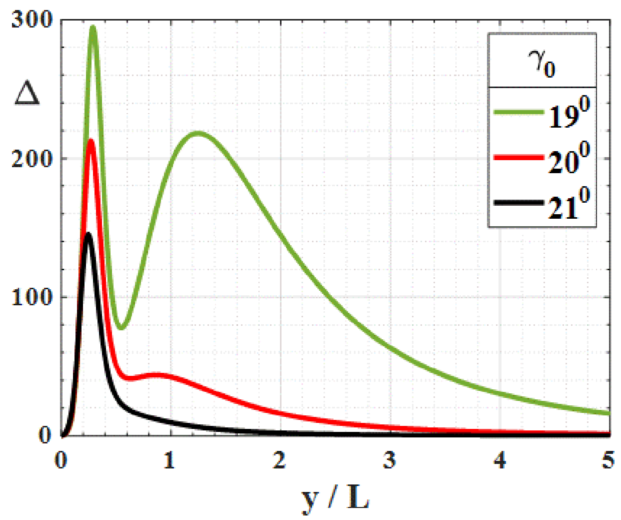

Figure 4. $\Delta - y/L$ diagram showing evaluation of the spectrum at different inclination angle of O-wave.

4. Discussion

When constructing a uniform asymptotic theory describing the processes of propagation and diffraction of electromagnetic waves (for example, radio pulses) in inhomogeneous anisotropic media in a short-wave approximation, geometric optics is the traditional method. Mathematical modeling of the features of the propagation of radio waves in the ionospheric plasma in the decameter range was carried out, taking into account artificial and natural inhomogeneities. The refractive index, one of the important parameters characterizing the equatorial ionosphere, has been derived for the first time. Second-order statistical moments of the temporal spectrum (broadening and displacement) of scattered ordinary and extraordinary radio waves propagating and scattering in the conductive turbulent plasma are investigated in the geometrical optics approximation.

Second-order statistical moments of the temporal spectrum: Correlation function and variance of the frequency fluctuations describing the broadening and shift of its maximum have been obtained for the arbitrary autocorrelation function of electron density fluctuations.

Numerical modeling was carried out for the spatial-temporal spectrum of electron density fluctuations containing anisotropic parameters characterizing turbulent ionospheric turbulent plasma: turbulent plasma velocity, anisotropy coefficient, and dip angle of stretched plasmonic structures relative to the lines of forces of the geomagnetic field. Investigations show that the anisotropy coefficient and dip angle of electron density inhomogeneities have a substantial influence on the broadening of the temporal power spectrum and the displacement of its maximum in scattered O- and E-waves in the equatorial ionosphere. A new double-humped effect has been revealed in the temporal spectrum of scattered O-waves in the equatorial ionosphere, contrary to the polar ionosphere. Diffraction effects have a substantial influence on the variance of the frequency fluctuations in the nonstationary turbulent plasma at longitudinal propagation, when the absorption is essential.

The relevance of the problem of studying the propagation of electromagnetic waves in the upper atmosphere is determined by the need to solve the problems of long-distance radio communication, radio navigation, radar location, as well as the problems of diagnosing the structure of the ionosphere propagation medium and also studying of the structure of an ionosphere—the upper atmosphere of Earth by methods of remote sensing and a radio tomography. Measurements of the statistical characteristics of scattered electromagnetic waves by satellite, ground-based radar systems, or meteorological-ionospheric stations give information about ionospheric turbulent plasma irregularities. Relevance of research is defined by the active use of electromagnetic waves of a short-wave band in the antenna equipment, for providing long-distance radio communication, radio navigation, a radar location, and also studying of the structure of an ionosphere—the upper atmosphere of Earth by methods of remote sensing and a radio tomography.

Conflict of Interest

There is no conflict of interest.

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